ASTRON 329/429, Fall 2017 – Problem Set 4

Due on Wed Nov. 15, by 5 PM in Alex Gurvich's mailbox.

Solve the following problems (no additional problems in Ryden).

I. Suppose the temperature T of a blackbody distribution is such that $kT \ll Q$, where $Q = 13.6$ eV is the ionization energy of hydrogen. What fraction f of the blackbody photons are energetic enough to ionize hydrogen? If $T = T_{\rm rec} = 3,740$ K, what is the numerical value of f?

II. (Ryden 8.3) Imagine that at the time of recombination, the baryonic portion of the universe consisted entirely of ^{4}He (that is, helium with two protons and two neutrons in its nucleus). The ionization energy of helium (that is the energy required to convert neutral He to He^+ is $Q_{\rm He} = 24.6$ eV. At what temperature would the fractional ionization of the helium be $X = 1/2$? Assume that the baryon-to-photon ratio $\eta = 5.5 \times 10^{-10}$ and that the number density of He^{++} is negligibly small. [The relevant statistical weight factor for the ionization of helium is $g_{\rm He}/g_e g_{\rm He^+} = 1/4$.]

III. CMB optical depth. We know from observations of the Lyα forest that the intergalactic medium is currently nearly fully ionized (otherwise, the Lyα forest would appear "black," i.e. there would be negligible transmitted flux). This implies that the intergalactic medium must have been reionized after recombination (likely through the action of the first stars and galaxies that produced ionizing photons). Detailed measurements of the CMB constrain the electron (Thomson) scattering optical depth to reionization,

$$\tau_{\rm reion} = \int_{t_{\rm recomb}}^{t_0} dt c \sigma_{\rm T} n_{\rm e}, \tag{1}$$

where $\sigma_{\rm T} = 6.65 \times 10^{-25}$ cm^2 is the Thomson cross section, $t_{\rm recomb}$ is the age of the universe at recombination, t_0 is the present time, and $n_{\rm e}$ is the electron number density. For simplicity, assume that the Universe is flat and matter dominated with $H_0 = 70$ km/s/Mpc and $\Omega_{\rm b} = 0.05$. Also assume for simplicity that the baryonic component consists of hydrogen only so that $n_{\rm e,0} \approx \rho_{\rm crit,0} \Omega_{\rm b}/m_{\rm p}$.

Consider a post-recombination history described by a step function such that the Universe is completely neutral before $z_{\rm reion}$ and fully neutral after that time (the "instantaneous reionization" approximation). The Planck satellite measured $\tau_{\rm reion} \approx 0.07$. Obtain an explicit analytic expression for $z_{\rm reion}$ as a function of $\tau_{\rm reion}$ and the other cosmological parameters under the simplifying assumptions of this problem. Evaluate numerically the instantaneous reionization redshift implied by Planck.

Note: This topic is discussed in Ryden's chapter 12, but you have all the knowledge needed to solve this problem already. You will learn more by solving it independently of the textbook.